Nail Down the Misconception:

$$\frac{a+b}{a+c} = \frac{b}{c}$$

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Note: This talk is based on the article with the same title published in MAA Innovative College Teaching Online Magazine:

What Profession Resembles Teaching ?

Teacher - Doctor

Connections between teaching and medical treatment

After so many years of teaching, I found that there are many connections between teaching and medical treatment. For example, when you don't feel good about a certain part of your body, you go to the doctor and describe your problem. The doctor would look at all the possible illnesses with your symptoms. He would use the method of exclusion to finalize his diagnosis. When we teach, we see a mistake made by students, we think how many possibilities there are for making this mistake and exclude them one by one until we find the right reason. Once we know the reason, we can correct the mistake efficiently just like the doctor who can give the right medicine to treat the disease.

(Quoted from an award winning high math teacher in Guangzhou, China)

What's Up With That?

There are some common mistakes students make repeatedly in Algebra class.

Can you think of one ?

A Mistake I Observed

Having taught Intermediate Algebra at college level for many years, I found that a quite number of students kept making this common mistake. Even in a Calculus class, this mistake is not uncommon in students' work. Whenever they have an algebraic expression containing like terms in the numerator and denominator, they simply cancel them out.

Eg. $\frac{\cancel{a}+b}{\cancel{a}+c}=\frac{b}{c}$; $\frac{\cancel{x^2}+3}{\cancel{x^2}-5}=-\frac{3}{5}$; $\frac{\cancel{2p^3}+3\cancel{p}+7}{\cancel{2p^3}+\cancel{p}+5}=2$; … …

I had been wondering why. Can we do something to correct the misconception effectively?

Let’s diagnose why students make this kind of mistakes

If a student makes this kind of mistake, what could be the possible reasons? What mathematical misconception(s) make the student not sense the mistake?

Form a group with your neighbors (2 or 3 people) to discuss the possible reasons. Each group will share your ideas produced from your discussion.

Based on our diagnosis, what can we do to address this problems?

Activities I did in my class

1) Students formed groups. Each group had three or four students.
2) Discuss the question: Is $\dfrac{a+b}{a+c} = \dfrac{b}{c}$

   given that b≠ c and neither a nor b is 0?
3) After 10-minute discussion, each group had a representative to report their answer with reasoning.

Step 1: Group Discussion

3) Reasoning for “Yes”: “a” is in the numerator and also in the denominator, by division, it can be cancelled out.

Reasoning for “No”: “a” can’t be cancelled because the operation sign between “a” and “c” and “a” and “b” are addition, not multiplication.

Students could not wait to hear my judgment. I told them that I would not tell them who is correct just yet.

Step 2: Working on the Worksheet

a	b	c	a + c	a + b	$\frac{a+b}{a+c}$	ac	ab	$\frac{ac}{ab}$	$\frac{b}{c}$
1	2	9							
2	3	1							
3	5	4							
4	6	3							
5	1	2							
6	2	4							
7	4	3							
8	3	1							
9	1	2							

Step 2: Working on the Worksheet

After 15 minutes, all the groups finished their work. I asked the groups one by one to report what they found. They all reported that

$$\frac{a+b}{a+c} \neq \frac{b}{c}$$

in all the cases of the table. I then asked them to make a conclusion based on the data.

Step 3: Prove the Conclusion Algebraically

1) Each group worked on the proof.

   (Giving the hint: Using Cross Multiply)

2) Ten minutes later, all groups explained their ideas of proof.

3) Writing the proof step by step on the blackboard:

   First, we suppose that $\frac{a+b}{a+c} = \frac{b}{c}$

Step 3: Prove the Conclusion Algebraically

Second, by cross multiplying both sides, we get

$$b\,(a + c) = c\,(a + b)$$

Third, distributing both sides, we have

$$ab + bc = ac + bc$$

Fourth, subtract bc from both sides, we get

$$ab = ac$$

Fifth, divide both side by 'a' (since $a \neq 0$), we get

$b = c$, which contradicts the condition

$$b \neq c$$

Step 3: Prove the Conclusion Algebraically

So clearly $\frac{a+b}{a+c} = \frac{b}{c}$

is only true algebraically if b = c.

Therefore $\frac{a+b}{a+c} \neq \frac{b}{c}$

which means that we can't cancel out "a" when given $\frac{a+b}{a+c}$

Step 4: Leting Students Think Further

Raise the following question:

What is the difference between the two algebraic expressions:

$$\frac{a+b}{a+c}$$

and

$$\frac{ac}{ab}$$

Step 4: Let Students Think Further: Making Connection to the Operation Rules

Students answered that in the first one the relationships between “a” and “c” and “a” and “b” are addition but in the second one the relationships between ‘a’ and ‘c’ and ‘a’ and ‘b’ are multiplication. Then I asked them: based on <u>the operation rule</u>, what is the order of operation for the first expression? “We do additions first and then divide”, some students answered.

Step 4: Let Students Think Further : Making Connection to the Operation Rules

One student responded: “Aha, now I see why I can’t cancel out ‘a’. Now it makes sense to me.”

I raised another question: How about $\frac{ac}{ab}$

Some students replied: “based on operation rule, when we do division and multiplication, we can do either operation first by the operation rule.” “Then what can we do about ‘a’ in this case?” I asked further. A number of students spoke out: “‘a’ can be cancelled by division”.

Step 5: Summarizing What We Did

I addressed a summary to my students: “As you can see, according to the data from the work sheet of table, the algebraic proof, and the operation rule, they all give us the same conclusion:

$$\frac{a+b}{a+c} \neq \frac{b}{c}$$

I used a red chalk to write this in a big size of letters on the board. Furthermore, I asked the groups to write a report reasoning why based on what we did in the class.

Assessment Questions

Determine if the following statements are true or false. Give your reasons.

a) $\frac{2\cancel{x}}{3\cancel{x}} = \frac{2}{3}$

b) $\frac{\cancel{2x}+1}{\cancel{2x}+2} = \frac{1}{2}$

c) $\frac{\cancel{3a}+5}{\cancel{3a}} = 5$

d) $\frac{12\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} = \frac{12}{x-1}$

e) $\frac{6\cancel{x}+5}{3x^{\cancel{2}}y} = \frac{11}{3xy}$

f) $\frac{x^{\cancel{2}}}{\cancel{x}+7} = \frac{x}{7}$

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The Teaching Approach in General

It is worth to mention that although we focused on a specific example, the method - having students work out numerical examples, then discussing the algebra, then looking at the differences between the situations where what the student wants to do is CORRECT versus where it is not, and finally practicing with some more sophisticated examples - is a promising one for other common student errors as well.